CANDIDATE'S NUMBER
(For the use of the Department only)

Tests
D-2
Math 30X
Supp. 1967

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DEPARTMENT
EDUCATION ALBERTA

HIGH SCHOOL AND UNIVERSITY MATRICULATION
EXAMINATIONS BOARD

DEPARTMENTAL EXAMINATIONS, 1967

SUPPLEMENTAL

MATHEMATICS 30X

Time — 3 hours.

DO NOT SPEND TOO MUCH TIME ON ANY ONE QUESTION.
READ THE QUESTIONS CAREFULLY.

Total Possible Mark — 163

All your work must be done in this booklet.
Do not write your name on this booklet.

Knott's Mathematical Tables supplied by the Presiding Examiner may be used. Candidates are NOT permitted to use slide rules.

Finished work in this booklet may be done in either pencil or ink (blue or black).

Do not put work which is to be read by the sub-examiner in the ROUGH WORK ONLY column.

Complete solutions for all questions should be shown.
Some marks may be obtained for partial solutions.

DO NOT FOLD THE BOOKLET.

Place your test booklet in the answer envelope in such a way that the **UPPER LEFT HAND CORNER OF THE BOOKLET** (Candidate's Number) shows through the **SLASHED CORNER** of the answer envelope.

BE SURE TO SEAL THE ENVELOPE.

CURRICULUM
LB
3054
C2
D419
1967a
math.30X
CURR HIST

(For use of Sub-examiners only)

Values

FOR ROUGH WORK
(No marks given for work
in this space)

2 1. Simplify with positive i

$$\frac{1}{x^{-1} + y^{-1}}$$

2 2. Rewrite with a rational denominator and simplify:

$$\frac{2 + \sqrt{3}}{3 - \sqrt{2}}$$

2 3. $\sin(-290)^{\circ}$ as a trigonometric function of a positive acute angle is

2 4. Use mapping notation to describe

$h = \{(x, y) \mid y^2 = x - 4,\ x, y \in R\}$

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Values

FOR ROUGH WORK
(No marks given for work in this space)

2 5. Convert 70° to radian measure.

2 6. If $\tan\theta = -\frac{5}{4}$, and θ is in standard position with terminal ray in the second quadrant find $\cos\theta$.

2 7. Write in standard form $a + bi$, $i = \sqrt{-1}$

$$\frac{1 - i^3}{1 - i}$$

Values

FOR ROUGH WORK
(No marks given for work in this space)

3 8. (a) Write in words the meaning of the symbol b(x; n, p).

2 (b) Evaluate b(2; 3, .5)

$x; n, p$

$= \binom{n}{x} p^{x} q^{n-x}$

$= \binom{3}{2} 0.5^{2} \times 0.5$

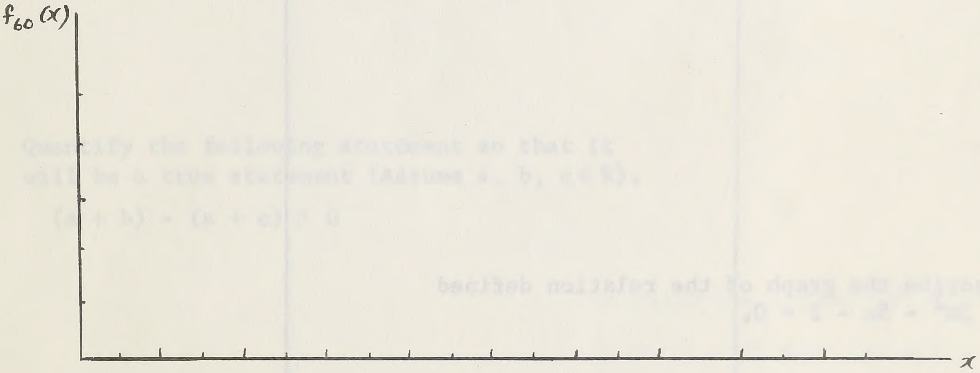
Values

9. The mathematics grades of 60 students obtained on a recent test are tabulated below, where x is the number of marks and $f_{60}(x)$ is the corresponding frequency of students.

x	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100
$f_{60}(x)$	0	2	4	2	9	14	15	12	2	0

3 (a) Find the mean grade of the 60 students listed.

3 (b) Construct a corresponding histogram for the data.



FOR ROUGH WORK
(No marks given for work in this space)

Values

FOR ROUGH WORK
(No marks given for work in this space)

3 10. Solve for n if $(n - 1)_2 = \binom{n}{2}$

2 11. Simplify: $\binom{9}{7} + \binom{9}{6}$

3 12. Describe the graph of the relation defined by $3x^2 - 5x - 2 = 0$.

Values

FOR ROUGH WORK
(No marks given for work
in this space)

3 13. Complete the following table:

P	Q	$\sim P$	$\sim P \wedge Q$	$\sim P \rightarrow Q$
T	T			
T	F			
F	T			
F	F			

2 14. If P = The book is mine, and Q = The book is green, write in words, the sentence that would represent $\sim P \leftrightarrow \sim Q$.

2 15. Quantify the following statement so that it will be a true statement (Assume a, b, $c \in R$).

$(a + b) - (a + c) > 0$

(OVER)

Values

FOR ROUGH WORK
(No marks given for work in this space)

4 16. If $\oplus$ is addition modulo 3, assuming the operation is associative, show clearly whether $\{0, 1, 2\}$ and $\oplus$ form a group.

3 17. State the Domain and Range of

$P = \{(x, y) \mid y = x^2 - 6, x, y \in R\}$

Values

FOR ROUGH WORK
(No marks given for work in this space)

2 18. If $f(x) = \frac{3x^2 - x}{x + 1}$, evaluate $f(x + 1)$

3 19. If $f(x) = \underline{x^2 + 4}$, find $f^{-1}(x)$

Values

5 20. Find the length of | PQ | formed by the intersection of

$A = \{(x, y) \mid y = 2x^2 + x - 6,\ x \in R\}$ and

$B = \{(x, y) \mid y = -3x,\ x \in R\}$, if P and Q are the points of intersection.

$y =$

FOR ROUGH WORK
(No marks given for work in this space)

$0 = 2x^2 + 4x - 6$

$= 2(x^2 + 2x - 3)$

$= 2(x + 3)(x - 1)$

$\therefore x = -3, 1$

$\therefore y =$

Values

FOR ROUGH WORK
(No marks given for work

1 21. (a) If one root of a quadratic equation is $2 + \sqrt{3}i$, and the coefficients are real, find the other root.

3 (b) State the equation in (a) above with integral co-efficients.

4 22. Determine the real values of k for which $kx^2 - kx + 2x + 1 = 0$ has real and unequal roots.

$b^2 - 4ac > 0$

$\boxed{K > 4+2\sqrt{3} \;\&\; K < 4-2\sqrt{3}}$

$(2-k)^2 - 4k > 0$

$4 - 4k + k^2 - 4k > 0$

$k^2 - 8k + 4 > 0$

$x = \frac{8 \pm \sqrt{64-16}}{2} = 4 \pm 2\sqrt{3}$

$4-2\sqrt{3}$ ① $4+2\sqrt{3}$

$\frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$\sqrt{64-16}$

Values

FOR ROUGH WORK
(No marks given for work in this space)

5 23. Find the equation in x with integral co-efficients each of whose roots will be 2 units less than the roots of $4x^2 - 2x - 5 = 0$.

[illegible]

$x = \frac{1 \pm \sqrt{1+20}}{4}$

$= \frac{1 \pm \sqrt{21}}{4}$

$\left(\frac{1+\sqrt{21}}{4} - 2\right), \left(\frac{1-\sqrt{21}}{4} - 2\right)$

$\left(x - \frac{\sqrt{21}-7}{4}\right)\left(x + \frac{7+\sqrt{21}}{4}\right) = 0$

$x^2 + \left(-\frac{\sqrt{21}-7}{4} + \frac{7+\sqrt{21}}{4}\right)x - \frac{21-49}{16} = 0$

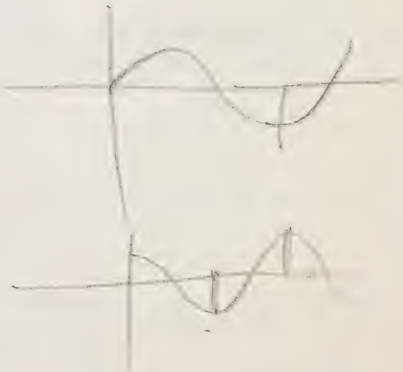
$x^2 + \frac{14}{4}x + \frac{27}{4} = 0$

$\frac{-7-\sqrt{21}}{2}$ $\frac{-7+\sqrt{21}}{2}$

3 24. Evaluate: $\sin \frac{3\pi}{2} - 2 \sin \frac{\pi}{6} + \frac{3}{\cos 3\pi}$

$-1 - \frac{2}{2} + \frac{3}{-1}$

$= -5$



Values

FOR ROUGH WORK
(No marks given for work
in this space)

5 25. Find the equation of the locus of the points $P(x, y)$ such that the distance from P to $Q(2, 4)$ is $\frac{2}{3}$ the distance from P to a line defined by $y = -1$.

4 26. Use the identity $\cos 2x = \cos^2 x - \sin^2 x$ to prove $\sin^2 \frac{\theta}{2} = \frac{1 - \cos \theta}{2}$

Values

FOR ROUGH WORK
(No marks given for work in this space)

4 27. Show that $(\cos\theta - \sin\theta)^2 - (\cos\theta + \sin\theta)^2$

$= -2\sin 2\theta.$

$\cos^2\theta - 1\sin 2\theta + \sin^2\theta - (\cos^2\theta + \sin 2\theta + \sin^2\theta)$

$2\sin\theta\cos\theta$

$= 1 - \sin 2\theta - 1 - \sin 2\theta$

$= \sin 2\theta$

$= \underline{-2\sin 2\theta}$

[illegible]

5 28. Use the formula sin(x + y) =

sin x cos y + cos x sin y

to find the exact value of cos 15° . (Leave your answer in surd form)

~~cos(45 - 30)~~

~~= cos 45~~

$\sin(30 + 15) = \frac{\sqrt{2}}{2}$

$= \sin 30\cos 15 + \cos 30\sin 15$

Values			FOR ROUGH WORK (No marks given for work in this space)
4	29.	If $z_1 = a + bi$ and $z_2 = c + di$, find in the form of $a + bi$, $\frac{z_1}{z_2}$	

$\frac{a+bi}{c+di} = \frac{(a+bi)(c-di)}{(c+di)(c-di)}$

$= \frac{ac + (b-d)i - bd i^2}{c^2 - (-d^2)}$

$= \frac{ac+bd + (b-d)i}{c^2+d^2}$

$a = \frac{ac+bd}{c^2+d^2}$

$b = \frac{b-d}{c^2+d^2}$

4 30. Find the three roots of $x^3 - 27 = 0$.

$x^3 - 27 = 0$

$(x-3)(x^2+3x+9)$

$x^3 = 27$

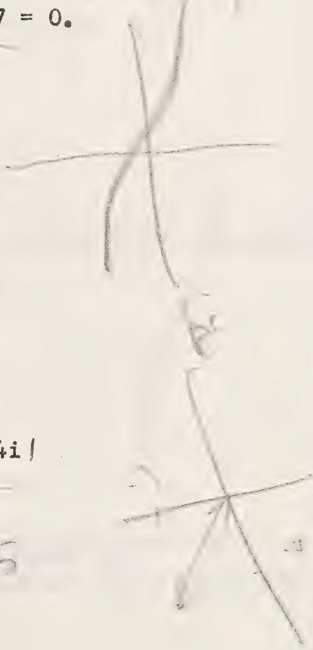
$x = +3$

$x = \frac{-3 \pm \sqrt{9-36}}{2}$

$x = \frac{-3 \pm \sqrt{27}\,i}{2}$

$-\sqrt{3}i$

$\sqrt{3}i$



3 31. (a) Find the value of $| - 3 - 4i |$

$\sqrt{9+16} = 5$

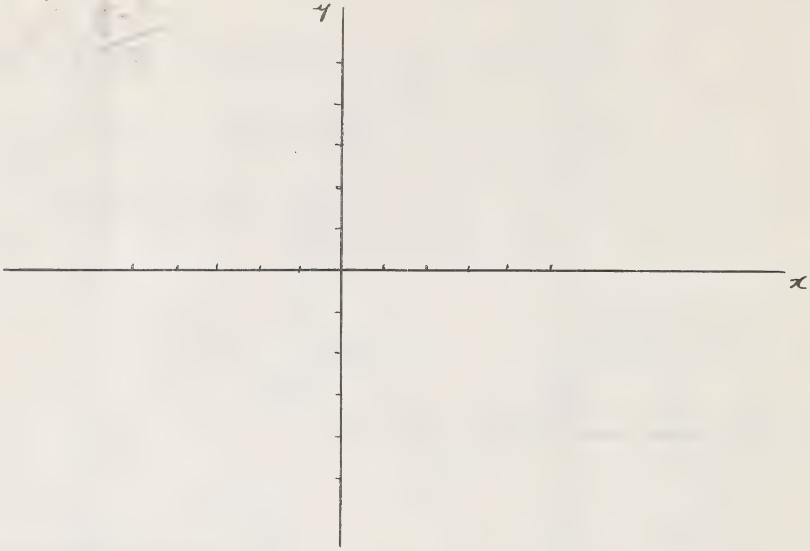
2 (b) State the Cartesian co-ordinates of the point P which represents the number $5 + 6i$.

(5, 6i)

Values

5 32. (a) Shade the region defined by:

$$\{(x, y) \mid y^2 \leq 3x + 2,\ x, y \in R\} \cap \{(x, y) \mid 2x^2 + 3y^2 \leq 48,\ x, y \in R\}$$



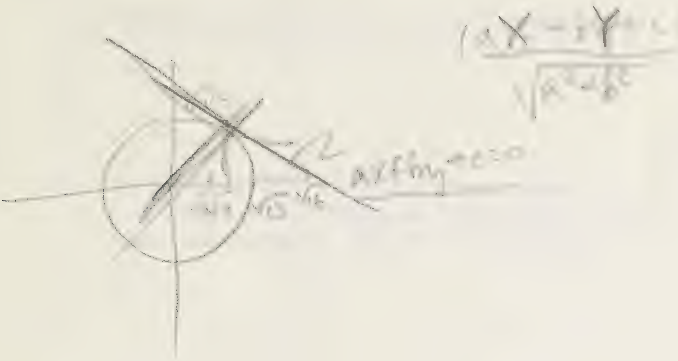
3 (b) State the co-ordinates focal points of $2x^2 + 3y^2 = 36$.

$$\frac{x^2}{18} + \frac{y^2}{12} = 1$$

FOR ROUGH WORK
(No marks given for work in this space)

Values

4 33. Determine the equation of the tangent through P(2, 3) on the circle defined by $x^2 + y^2 = 13$.



6 34. Prove by mathematical induction that

$$\sum_{a=1}^{n} 2^a = 2(2^n - 1)$$

FOR ROUGH WORK
(No marks given for work
in this space)

Values

3 35. (a) Find in simplified form the general term of $\left(\sqrt{a}+\frac{2}{a}\right)^{10}$

3 (b) Find if there is a term containing a^{0} in the expansion of $\left(\sqrt{a}+\frac{2}{a}\right)^{10}$

3 (c) Which term in the expansion of $\left(\sqrt{a}+\frac{2}{a}\right)^{10}$ contains a^{2}?

$$ {}^{10}C_{r}\,(a^{\frac{1}{2}})^{10-r}\,(2a^{-1})^{r} $$

$$ a^{5-r/2}\quad a^{-r} $$

$$ 5-\tfrac{3}{2}r = 2 $$

$$ -\frac{3}{2}r = -3 $$

$$ r = 2 $$

$$ {}^{10}C_{2}\,(a^{1/2})^{8}\,(a^{-1})^{2} $$

$$ a^{4}(a^{-2}) $$

$$ a^{2} $$

FOR ROUGH WORK
(No marks given for work in this space)

Values

FOR ROUGH WORK
(No marks given for work in this space)

36. A coin is tossed until a head appears, or until it has been tossed three times.

2 (a) State a sample space for the experiment.

4 (b) Given that the head does not occur on the first toss, what is the probability that the coin is tossed three times?

3 37. (a) How many distinct "words" can be made using all the letters of "mathematics"?

3 (b) In how many of the above words does (math) ∪ (sit) appear?

$A \cup B =$ [illegible]

Values

FOR ROUGH WORK
(No marks given for work in this space)

4 38. A hockey player scored 40 goals in the last 60 games he played, find the probability that he will score 2 goals in the next 5 games he plays.

$\frac{40}{60}$ [illegible]

(15)

39. A club consists of 20 members. If a delegation of four members is to be chosen at random to prepare and deliver a brief what is the probability that:

3 (a) The delegation consists of the president and three other members?

2 (b) The delegation consists entirely of the four members of the executive? (President, Vice-President, Secretary, Treasurer)

Values

4 40. If $P(A) + P(A') = 1$ and $P(B) + P(B') = 1$, $A, B \in S$

develop an expression for $P(A' \cap B)$ in terms of $P(A)$, $P(B)$ + $P(A \cap B)$.

FOR ROUGH WORK
(No marks given for work
in this space)

Values

FOR ROUGH WORK
(No marks given for work in this space)

41. Given the following data: -6, -3, 1, 5, 7, 12, 16, 20, 20, 24, 30, 31, 36.

1 (a) Determine the median score.

4 (b) Determine the quartile values of the data.

2 (c) Determine the interquartile range of the data.

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